Embedded Implicit Range-Kutta Nystrum Method for Solving Second Order Differential Equations

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Abstract: An embedded diagonally implicit Range-Kutta Nystrom (RKN) method is constructed for the integration of initial value problems for second order ordinary differential equations possessing oscillatory solutions. This embedded method is derived using a three stage diagonally implicit Runge-Kutta Nystrom method of order four within which a third order three stage diagonally implicit Runge-Kutta Nystrom method is embedded. We demonstrate how this system can be solved and by an appropriate choice of free parameters, we obtain an optimized RKN (4, 3) embedded algorithm. We also examine the intervals of stability and show that the method is strongly stable within an appropriate region of stability and is thus suitable for oscillatory problems by applying the method to the test equation $y'' = -\omega^2 y$, $\omega > 0$. Necessary and sufficient conditions are given for this method to possess non-vanishing intervals of periodicity, for the fourth order method. Finally we present the coefficients of the method optimized for small truncation errors. This new scheme is likely to be efficient for the numerical integration of second order differential equations with periodic solutions, using adaptive step size.

Keywords: Initial value problems; diagonally implicit; second order equations; embedded Runge-Kutta Nystrom; step size control